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Research Article

Analysis of the Adaptation of “Spar” Points in Tirana with Graph Theory and Elimination Algorithm

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Abstract

The network's topological structure is an important determinant in analyzing the availability and efficiency of the network with respect to accessibility and service delivery in urban settings. In this study, the “Spar” retail network in Tirana was used to conduct a graph theory approach to model network availability and distribution. In graph theory representation, the “Spar” retail outlets are denoted by nodes while the connecting roads are represented by directed and weighted graphs determined using Google Maps. The output or result is presented through the critical matrix process called the elimination algorithm technique used in determining minimum paths represented in weighted graphs. By eliminating matrixes in sequence, the technique was used to calculate the minimum path between certain “Spar” retail networks and demonstrates how “Spar”-QTU and “Spar”- Bulevardi Kashar are related. The study demonstrates how using graph theory in retail distribution networks is an important technique in analyzing logistics and network availability. It is recommended that the technique has the ability to be used in analyzing retail and service networks operating in an urban setting.

Keywords: graph, algorithm, minimum distance, Spar points, networks, graph theory

JEL Codes: C61, C63, L81

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Introduction

This study analyzes the geographical distribution of the points of the "Spar" market network in Tirana and the relationships between them using graph theory. The main goal is to find the minimum distance between the points through the elimination algorithm, which is one of the classic techniques used in weighted graphs.

The "Spar" points are distributed in different areas of Tirana and offer a wide range of services to consumers. Through graph theory, these points are modeled as nodes, while the road distances between them represent the weighted angles of the graph.

Graph theory studies the structure of connections between entities and provides efficient methods for analyzing networks, including:

1. Nodes (points): A point is the basic element of a graph and represents a specific entity or object being studied. It can represent anything from cities on a map to pages or users in a social network.
2. Links (angles): An angle is the connection between two points in a graph. Angels can either be directed, undirected, weighted, or unweighted, which depends on the nature of the relationship between the points.
3. Directed and undirected graphs: Directed vs. Undirected Graphs. A directed graph contains angles with a specific direction, which symbolizes the flow of a relation. An undirected graph has graph angles without direction, symbolizing an equal relation.
4. Weighted and unweighted graphs: Weighted and Unweighted Graphs: A graph where every connection has a certain weight associated with it which provides extra information concerning the relation, for example, distance, weight, and so on. An unweighted graph contains all connections of equal value without any extra information. [3][8]

Graph theory is found in many fields, from computer science and engineering, to biology, economics, and road navigation. In this study, it is used to analyze the distribution and accessibility between Spar points.[1][4][9][7].

Literature Review

To describe shortest path problems, these studies should be placed in the context of graph theory. The most widely used methods in solving shortest path problems in the literature include Dijkstra's Algorithm, introduced in [9], and Floyd-Warshall algorithm, presented in [8], in the context of graph theory. Both methods work in the context of weighted graphs, in particular those with non-negative edges.[6]

Graph theory-based techniques have found many successful usages in transport and logistics. Network modeling has been especially successful in analyzing issues of accessibility and route optimization in urban areas. Nikolla and Boci [7] used graph theory for optimizing postal transport networks and demonstrated that mathematical models of transportation have a great impact on optimizing distribution.[9] successfully utilized GIS network optimization methods for planning service within big cities.

The elimination algorithm is the matrix alternative for the shortest-path analysis. In comparison to Dijkstra's algorithm, which is concerned with the choices of the individual source, or the Floyd-Warshall algorithm, which finds the solution for all pairs shortest paths problem, the elimination algorithm is concerned with the progressive reduction of the distance matrix. This property makes the elimination algorithm appealing for instances where the central focus is on the pairs of nodes, and an explicit step-by-step solution process is desired.

Methodology

1. Data Collection

In this study, the data were used on road distances between Spar retail points located in different areas of Tirana. The distances were all obtained by using Google Maps and represent actual driven distances. Because of one-way streets and traffic regulations, the distance between two locations is not necessarily the same in both directions. Therefore, the retail network needs to be modeled as a directed graph.[5]

2. Modeling with Graphs

The Spar retail outlets are represented by vertices (nodes), while the direct connections along the roads between the outlets represent directed edges. The weights on the edges reflect the distances between the endpoints along the roads, in kilometers. If there is no connection-quite apart from a detour-between two places, the distance matrix is given the value ∞ at this position. Such modeling makes the Spar retail network clear and mathematically well-set.

3. Elimination

To calculate the minimum distances for the selected Spar locations, the elimination algorithm is run for the original distances A. The rule followed for the updating process in the original distance A after each step is given by the equation

$$a_{ij}' = \min(a_{ij}, a_{in} + a_{nj})$$

Once the matrix has been updated, a row and a column are eliminated due to the nature of the matrix. The process is repeated until a matrix of size 2 by 2 is obtained. The nondiagonal entries in the resulting matrix provide the shortest distances between the nodes.[2][8]

4. Strengths and Weaknesses of the Method

One of the greatest strengths of the elimination algorithm is in its methodical approach, where the algorithm is extremely useful for finding minimal paths without having to enumerate manually all possible paths in the process. This is particularly useful in medium-sized graphs, whose characteristics can be seen in the Spar Tirana network.

However, there are limitations to the method as well. As the size of the network increases, the complexity of the matrix operations increases, thereby making the elimination algorithm less efficient for large networks than Dijkstra or Floyd-Warshall algorithms. Moreover, the proposed method is primarily concerned with the problem of distance minimization and does not consider the effects of dynamic factors such as traffic congestion.

5. Justification of the Method Choice

The reason why the elimination method was selected for this paper is that it offers a thorough framework within which the calculation of minimum distances between individual Spar retail outlets can be assessed. The size of the problem and the fact that it attempts to capture the impact of the network and the effects of the indirect routes, from the point of view of overall accessibility, makes the method particularly relevant to this paper.

Findings and Interpretations

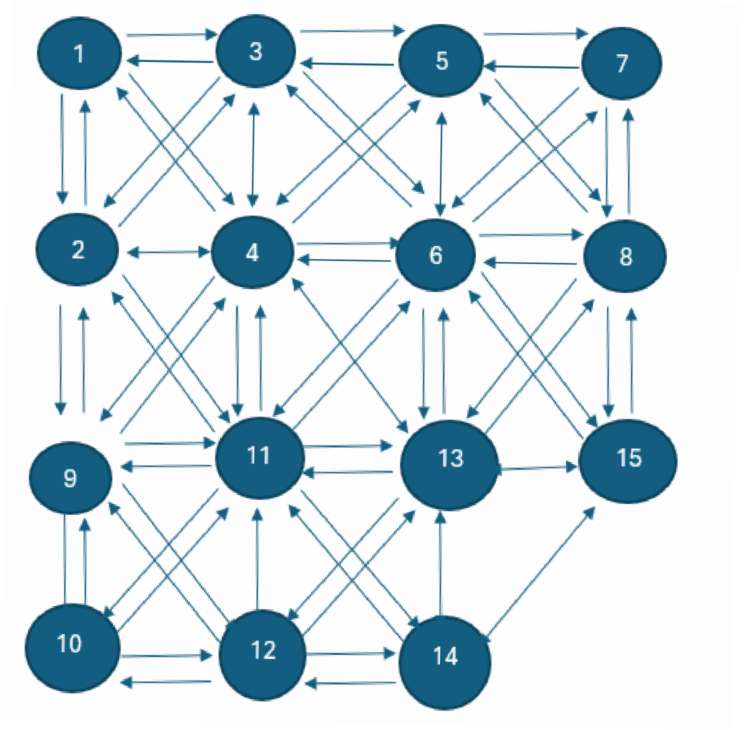
The data collected show the road distances between all 15 Spar points in Tirana that are listed below.

1. Spar – Qtu
2. Spar – Bulevardi Kashar
3. Spar – Street Korabi
4. Spar – Street Don Bosko
5. Spar – Condor Center
6. Spar – Street Him Kolli
7. Spar – Street e Bogdaneve
8. Spar – Street e Durresit

9. Spar – Street Memo Meto
10. Spar – Street e Dibres
11. Spar – Komuna e Parisit
12. Spar – Street Perlat Rexhepi
13. Spar – Street Ismail Qemali
14. Spar – Street Dervish Hima
15. Spar – Street Elbasani

The image shows all the connections between every Spar point.

Figure 1 - Directed Graph Representation of the Spar Network in Tirana



Source: Data from the author

Distances between Spar stores in Tirana

Figure 2 - The distance between every Spar Shop in Km

From\ to	QTU	Bulevardi Kashar	Street Korabi	Street Don Bosko	Condor Center	Street Him Kolli	Street Bogdaneve	Street e Durresit	Komuna e Parisit	Street Memo Meto	Street Ismail Qemali	Street Elbasanit	Street Dibres	Street Perlat Rexhepi	Street Dervish Hima
QTU	0	7,6	4,3	5,5											
Bulevardi Kashar	7,3	0	12	11					14	14					
Street Korabi	5,5	11	0	3,9	6,8	8,1									
Street Don Bosko	5,4	11	3,9	0	2,5	2,7			3,7	3.2	3,6				
Condor Center			6,5	3,2	0	1,9	2,1	3,8							
Street Him Kolli			8,5	2,8	1,9	0	2,1	2,5	2.5		1,7	7,7			
Street Bogdaneve					1,3	1,5	0	2,7							
Street Durresit					3,4	2,2	0,7	0			2	8,1			
Komuna e Parisit		15		3,4		2.3			0	4,9	2,4		3,6	1,9	2,7
Street Memo Meto		13		2.4					4,5	0			1,6	3	
Street Ismail Qemali				3,6		2		2.1	2,8		0	6		0,9	0,6
Street Elbasanit						9,6		7,7			6	0			5,9
Street Dibres									4,6	1,4			0	2,8	
Street Perlat Rexhepi									1,9	4	1,5		2,7	0	2,5
Street Dervish Hima									2,8		0,6	5,9		1,2	0

Source: Data from authors

Based on the directed graph representation of the Spar network, the next step involves converting all connections and road distances into a structured numerical format. For this purpose, the initial weighted matrix A is constructed, where each entry represents the direct distance between two locations or the symbol ∞ when no direct connection exists. This matrix serves as the starting point for the elimination algorithm, through which successive matrices A_1, A_2, A_3, … are generated by updating each element according to the substitution rule a_ij^'=min(a_ij, a_in+a_nj).

Each resulting matrix reduces the graph's order and progressively refines the minimum path estimates until the final 2×2 matrix is obtained.

$$A=\begin{vmatrix}
0 & 7{,}6 & 4{,}3 & 5{,}5 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
7{,}3 & 0 & 12 & 11 & \infty & \infty & \infty & \infty & 14 & \infty & 14 & \infty & \infty & \infty & \infty \\
5{,}5 & 11 & 0 & 3{,}9 & 6{,}8 & 8{,}1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
5{,}4 & 11 & 3{,}9 & 0 & 2{,}5 & 2{,}7 & \infty & \infty & 3{,}2 & \infty & 3{,}7 & \infty & 3{,}6 & \infty & \infty \\
\infty & \infty & 6{,}5 & 3{,}2 & 0 & 1{,}9 & 2{,}1 & 3{,}8 & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 8{,}5 & 2{,}8 & 1{,}9 & 0 & 2{,}1 & 2{,}5 & \infty & \infty & 2{,}5 & \infty & 1{,}7 & \infty & \mathbf{7{,}7} \\
\infty & \infty & \infty & \infty & 1{,}3 & 1{,}5 & 0 & 2{,}7 & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & 3{,}4 & 2{,}2 & 0{,}7 & 0 & \infty & \infty & \infty & \infty & 2 & \infty & 8{,}1 \\
\infty & 13 & \infty & 2{,}4 & \infty & \infty & \infty & \infty & 0 & 1{,}6 & 4{,}5 & 3 & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1{,}4 & 0 & 4{,}6 & 2{,}8 & \infty & \infty & \infty \\
\infty & 15 & \infty & 3{,}4 & \infty & 2{,}3 & \infty & \infty & 4{,}9 & 3{,}6 & 0 & 1{,}9 & 2{,}4 & 2{,}7 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4 & 2{,}7 & 1{,}9 & 0 & 1{,}5 & 2{,}5 & \infty \\
\infty & \infty & \infty & 3{,}6 & \infty & 2 & \infty & 2{,}1 & \infty & \infty & 2{,}8 & 0{,}9 & 0 & 0{,}6 & 6 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 2{,}8 & 1{,}2 & 0{,}6 & 0 & 5{,}9 \\
\infty & \infty & \infty & \infty & \infty & 9{,}6 & \infty & 7{,}7 & \infty & \infty & \infty & \infty & 6 & 5{,}9 & 0
\end{vmatrix}$$

The initial distance matrix was transformed step by step using the elimination algorithm. At the end of the algorithm, the minimum distances matrix is obtained:

$$\mathbf{A_{13}}= \begin{vmatrix} 0 & 7,6 \\ 7,3 & 0 \end{vmatrix}$$

- **Minimum distance from $X_1$to X_2** (Spar – Qtu to Spar – Bulevardi Kashar)**:**

$$m_{12} = 7.6\ \text{km}$$

- **Minimum distance from $X_2$to X_1** (Bulevardi Kashar –Spar – Qtu to Spar)**:**

$$m_{21} = 7.3\ \text{km}$$

These values represent the shortest possible paths according to the graph modeling and the input data.

The analysis starts from the initial weighted distance matrix and includes direct road distances between Spar locations, according to successive reductions made by the algorithm. In this process, a number of intermediate matrices are produced as part of distance value updates and rows and column removals. The final reduced matrix **A₁₃** indicates the least distances between the selected Spar points. Specifically, it is seen that the minimum distance between Spar–QTU and Spar–Bulevardi Kashar involves indirect connections of intermediate locations of Spar, without having a direct route from one to another.[10] This very important result proves that the network structure usually has a big impact on the minimum distances; therefore, networks could reach higher efficiency using intermediate points instead of direct connections. These nodes generally the travel distance and improve connectivity between more distant locations. In a practical sense, such information will be relevant for logistics planning and route optimization as strategic paths are given by them that enhance the efficiency of distribution.

The results reinforce that the elimination algorithm captures the minimum distance relations that characterize the Spar retail network. Interpretation is stressed over the exhaustive numerical presentation here in order to give a better idea of the spatial organization and functional efficiency of the network.

Conclusions

The given research also gave a quantitative assessment of the spatial arrangement of the 'Spar' retail chain in Tirana using a graph approach with a direction and weightage. The algorithm of removal of shortest paths using the actual distance on the Road Network was employed in the research to prove the shortest paths in a chain of retail stores are not necessarily the shortest paths between the nodes in the graph approach.

One of the major findings emerging from this analysis is that the network itself has a major role to play in terms of accessibility and efficiency. In this case, it was evident that the shortest path from Spar–QTU to Spar–Bulevardi Kashar covers the routes through the Spar intermediary stations. This highlights that there are a number of nodes that play a major role as connectors to boost the network's efficiency, thereby minimizing the routes between the far-off outlets.

The use of the elimination algorithm worked well in bringing out such properties of the structure. Contrary to the shortest path algorithm that targets a source vertex or pairs of vertices generally in graph theory, the algorithm used in the process enabled the gradual reduction of the matrix of distances such that the impact of vertices in between on the shortest paths was clearly noticed in the process.

From a practical perspective, it should be noted that these test results have implications relating to logistics and distribution because they indicate that network routing heuristics have a better optimum than straightforward distance calculations. Overall, it has been made clear that graph models coupled with methods of deletion from a matrix have an application within the context of decision support for analysis of store and service networks.

The study showed that graph theory is a powerful tool in analyzing the distribution of retail outlets and the distances between them. By applying the elimination algorithm on Spar outlets in Tirana, the minimum distances between the selected outlets were identified.

The method is efficient for further analysis such as:

- optimization of distribution networks,
- logistics planning,
- study of geographical accessibility,
- modeling of service networks in the city.

This analytical process can also be applied to other retail, transport or service networks.

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